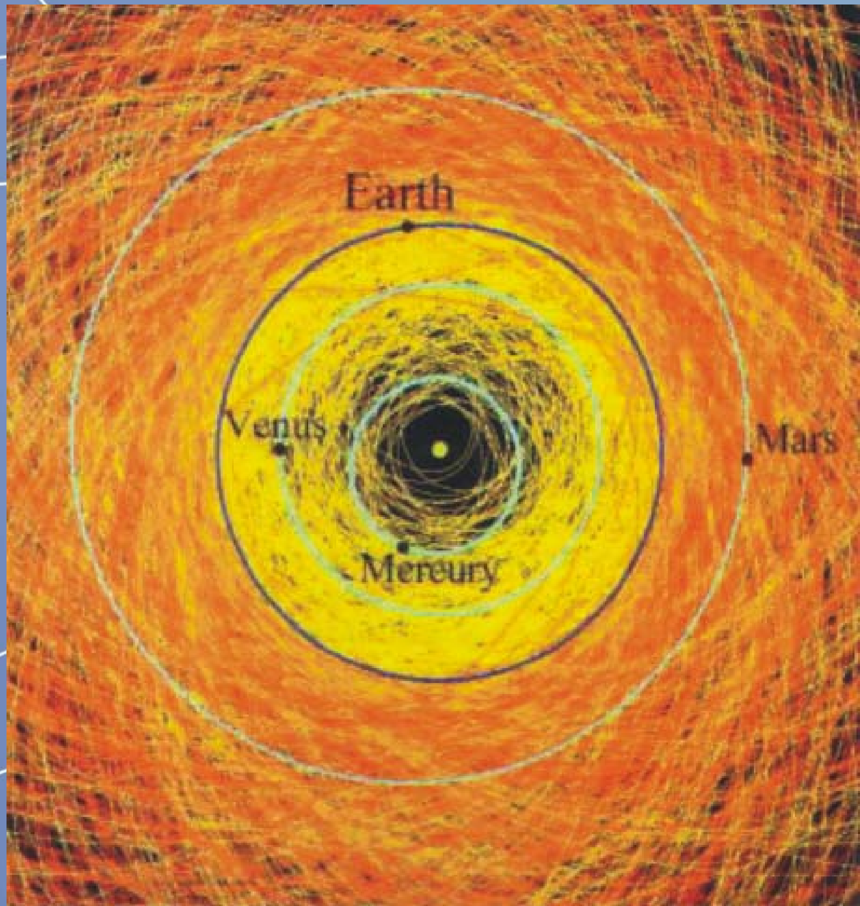
# Fuzzy Characterisation of Near-Earth-Asteroids

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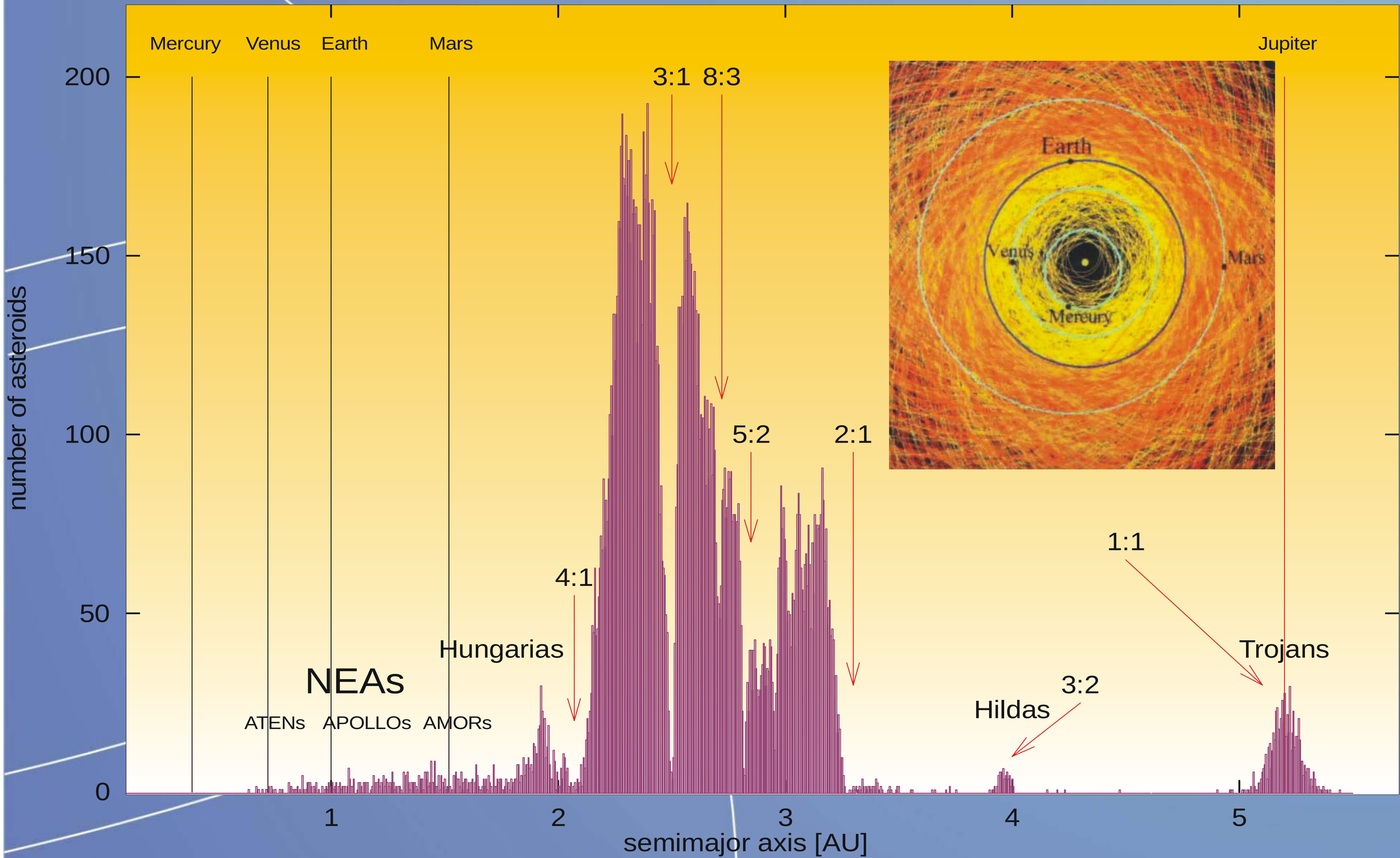
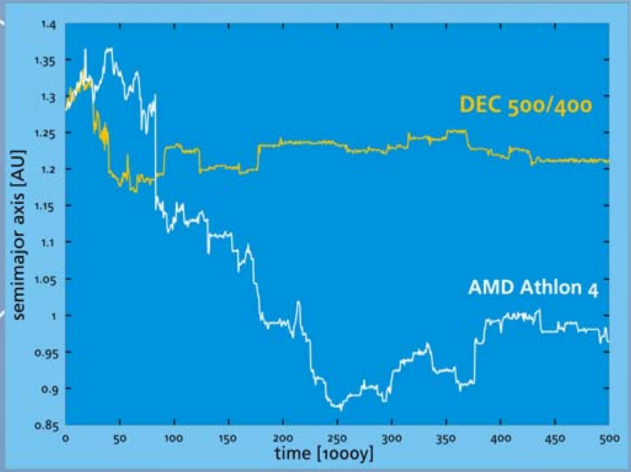
# Introduction



# NEA Dynamics

- NEAs have many close encounters with the inner planets
- close encounters result in drastic changes of orbital elements
- → NEAs have highly chaotic orbits over long time scales

- single object/single integration for long time scales makes no sense in chaotic systems
- → statistics




## NEA Classification

- Two important classifications for NEAs:

**Shoemaker et al. (1979)**

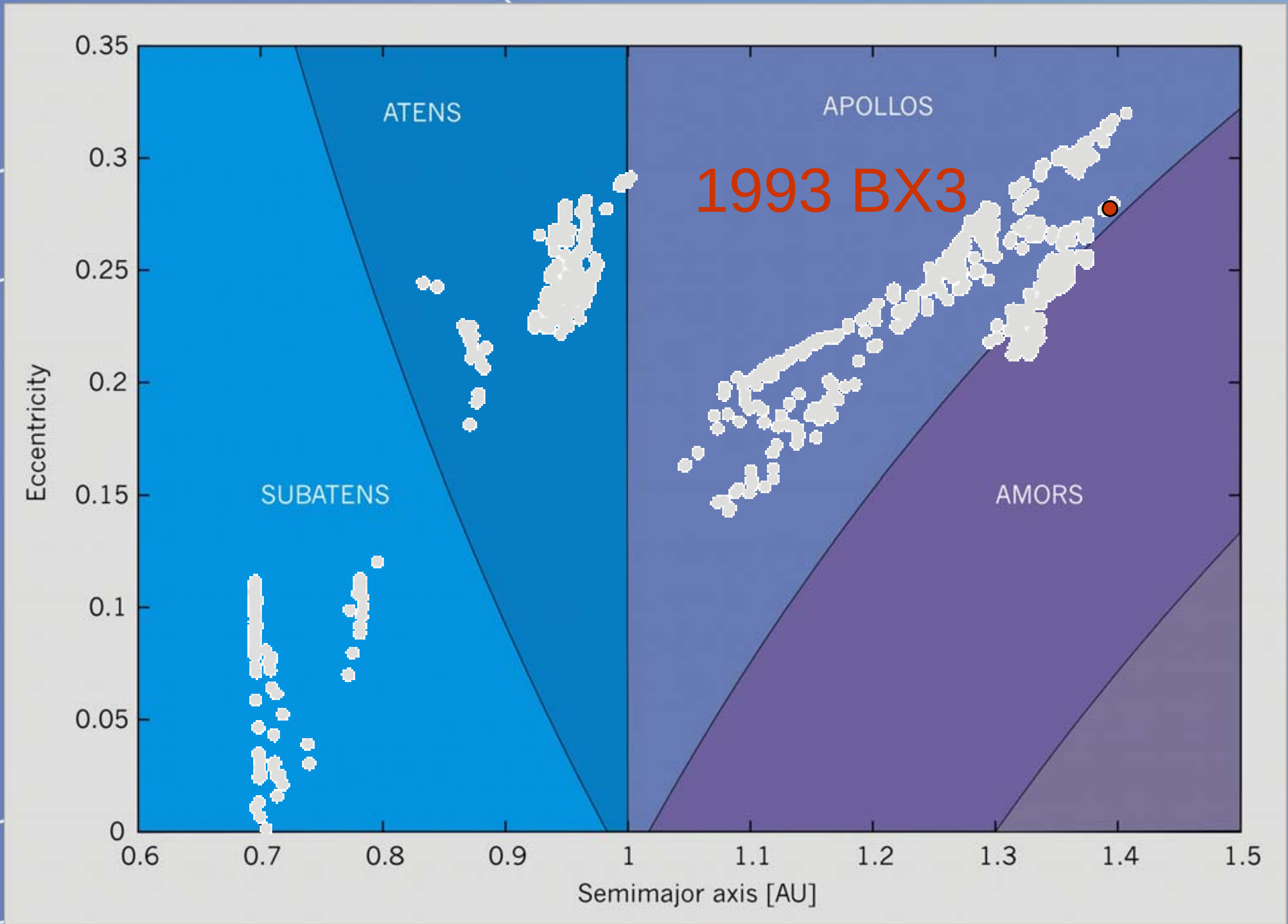
- Aten :: Apollo :: Amor class
- classification parameters:
  - a, e (crossing behaviour)

**Milani et al. (1989)**

- 7 classes
- dynamical behaviour:
  - encounters, resonances

# NEA Dynamics

• Chaotic orbits + long time scales = problems with statistics!




„Border Crossing Number"

(how many times does the NEA crosses a border of the Shoemaker classification in the a-e plane)

# NEA Dynamics

•How much time does a mean NEA spend in its initial group?

• **69.86 %** (Shoemaker et al. classification) *(Dvorak & Freistetter 2001, Freistetter 2006)*

• **65.72 %** (Milani et al. Classification) *(Milani et al 1989)*

• Classic NEA classifications are not applicable for long time scales („mixing“)

• → new methods

• Characterisation of NEA dynamics with **Fuzzy Logic**

# Fuzzy Logic

- developed 1965 by L.A. Zadeh
- Fuzzy sets are an extension of classical sets
- classical sets: an object either is or is not a memer of a group

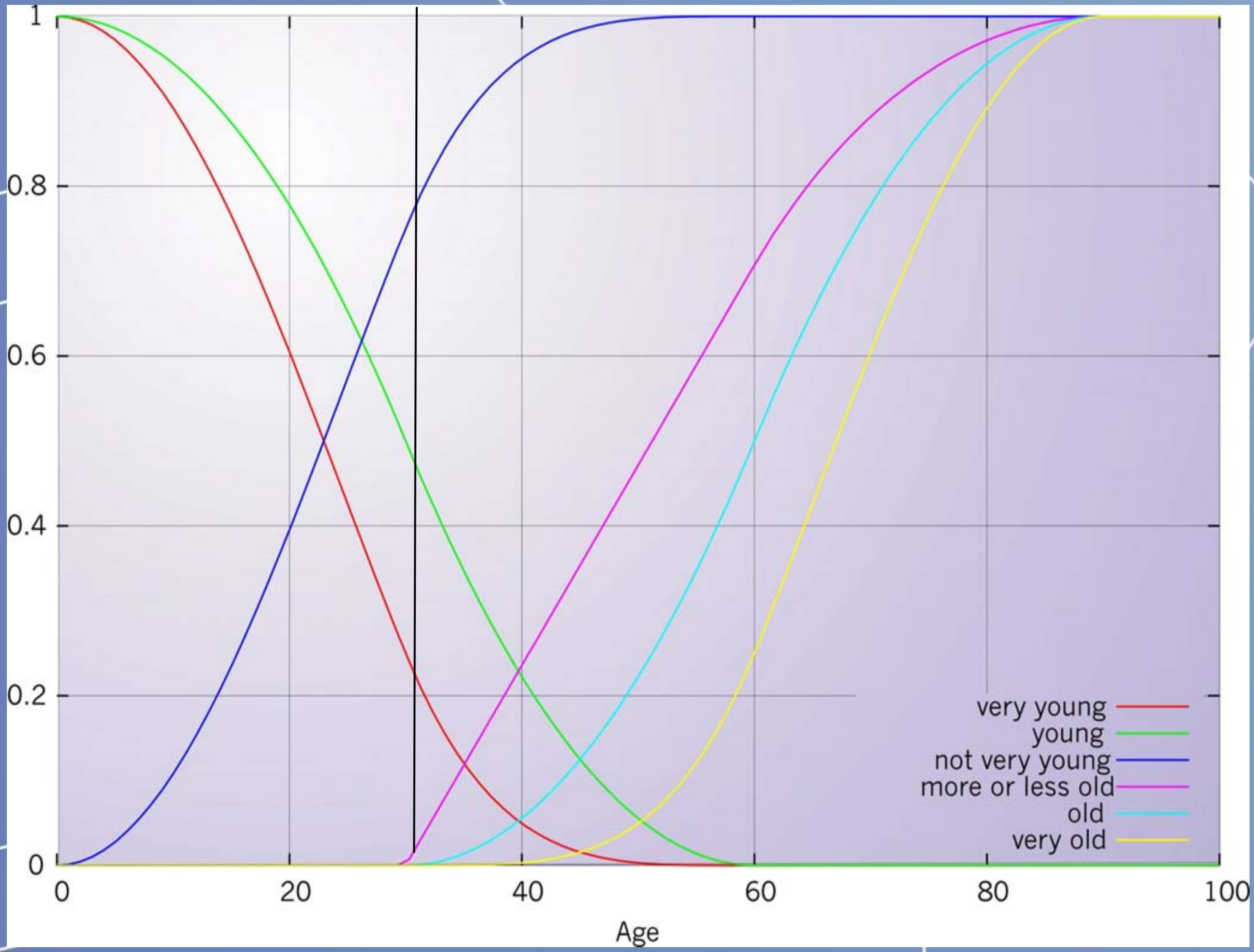
- → characteristic function: two values
- characteristic function of fuzzy sets can have all values between 0 and 1

$$f_A(x) = 1 \Leftrightarrow x \in A$$
$$f_A(x) = 0 \Leftrightarrow x \notin A$$

→ membership function $\mu_A(x)$ defines to which „degree“ x belongs to the fuzzy set A

# Fuzzy Logic

• Example: the age of a person




**Age: 30 years**

- very young: 25%
- young: 50%
- not very young: 75%
- more or less old: 2%
- old: 0%
- very old: 0%

It is possible to be a member of multiple groups with different degree!

# Fuzzy NEA classification

1. Definition of classes
2. Definition of membership functions
3. Classification
4. Analysis

Definition of classes

- NEAS can collide with inner planets → define classes to describe this properties
- as parameter for the membership function and the classification the number of close encounters is used.
- additional group to describe the mixing/motion in a-e plane

# Fuzzy NEA classification

## Definition of classes

- G1: NEAs that show almost no mixing
- G2: NEAS that can collide with Venus
- G3: NEAS that can collide with Earth
- G4: NEAS that can collide with Mars

## Parameters

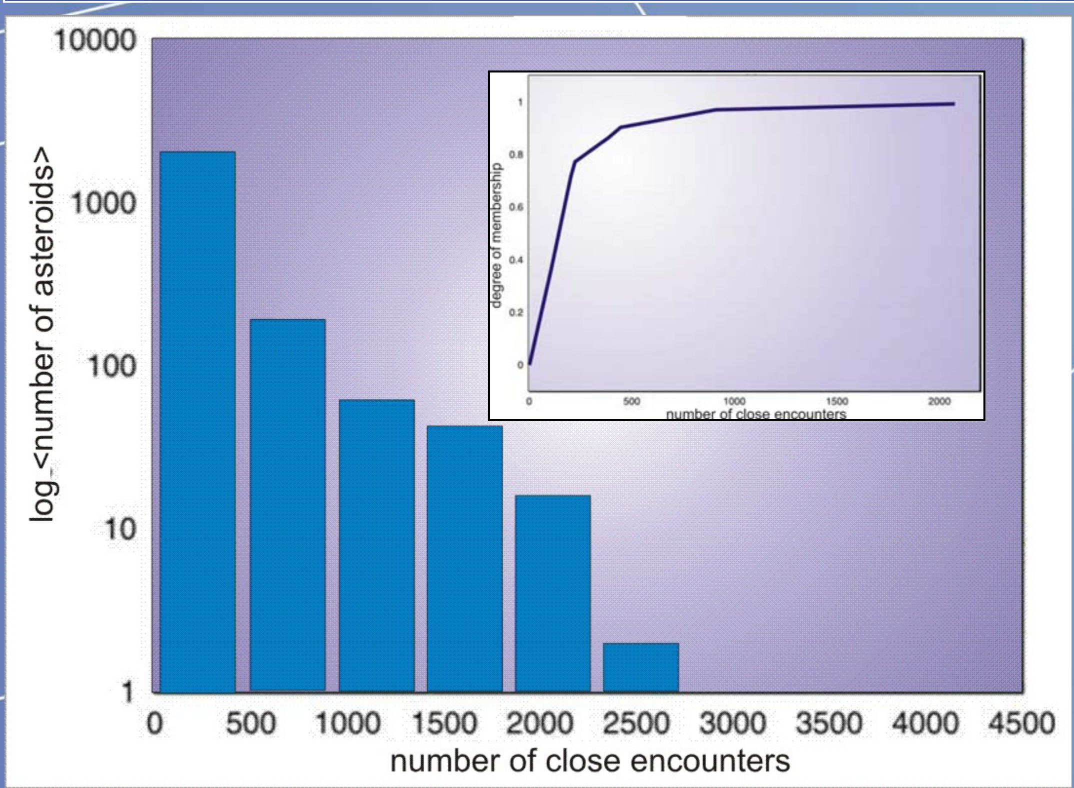
- G1: Border Crossing Number
- G2: close encounters with Venus
- G3: close encounters with Earth
- G4: close encounters with Mars

# Fuzzy NEA classification

## Membership Functions

• Investigate the statistical distribution of the parameters

• (numerical integration of the real NEAs for 500,000 years)




## G3

• convert distribution into membership function

• the more close encounters, the larger the degree of membership to G3

# Fuzzy NEA classification

| Name | a | e | i | G1 | G2 | G3 | G4 |
|---|---|---|---|---|---|---|---|
| 2000GD2 | 0.757928 | 0.4765 | 32.14639 | 1 | 0.7585 | 0.9407 | 0 |
| 2000HB24 | 0.815912 | 0.4302 | 2.669175 | 1 | 0.9852 | 0.9895 | 0 |
| 2000HO40 | 0.743917 | 0.5241 | 5.981537 | 1 | 0.9951 | 1 | 0 |
| 2000LG6 | 0.916167 | 0.1121 | 2.829914 | 0.0164 | 0 | 0.9973 | 0 |
| 2000NL10 | 0.914292 | 0.8171 | 32.51204 | 1 | 0.3271 | 0.436 | 0.0147 |
| 2000OK8 | 0.984809 | 0.2211 | 9.985352 | 0.7272 | 0 | 0.9299 | 0.6993 |
| 2000PH5 | 1.000608 | 0.2301 | 1.71175 | 0.1708 | 0.9962 | 0.9792 | 0 |
| 2000PJ5 | 0.8727 | 0.3736 | 51.18267 | 1 | 0.9742 | 0 | 0 |
| 2000QP | 0.847462 | 0.4631 | 34.7466 | 0.9008 | 0.9559 | 0 | 0 |
| 2000RH60 | 0.825892 | 0.5513 | 19.64366 | 1 | 0.9403 | 0.9191 | 0 |
| 2000RN77 | 0.951245 | 0.3184 | 16.0945 | 0.8264 | 0.9503 | 0.7779 | 0.7791 |
| 2000SG344 | 0.977357 | 0.0669 | 0.109731 | 0.0113 | 0.9929 | 0.9972 | 0 |
| 2000SP43 | 0.811372 | 0.4669 | 10.35569 | 1 | 0.9558 | 0.9378 | 0 |
| 2000SY2 | 0.858743 | 0.6426 | 19.23634 | 1 | 0.7415 | 0.7198 | 0.8302 |
| 2000SZ162 | 0.929366 | 0.1674 | 0.896554 | 0.2561 | 0.0126 | 0.9818 | 0.7287 |
| 2000UH11 | 0.870267 | 0.4223 | 32.21665 | 1 | 0.7179 | 0.6956 | 0 |
| 2000UK11 | 0.884688 | 0.2482 | 0.776174 | 0.9256 | 0.4127 | 0.9816 | 0.4049 |
| 2000UR16 | 0.903661 | 0.4387 | 11.74411 | 1 | 0.9341 | 0.8863 | 0 |
| 2000WC1 | 0.879512 | 0.2626 | 17.40797 | 0.8512 | 0.9356 | 0.8146 | 0 |
| 2000WO107 | 0.911347 | 0.7806 | 7.784174 | 1 | 0.9192 | 0.91 | 0.9963 |
| 2000WP19 | 0.854492 | 0.2886 | 7.678904 | 0.6033 | 0.9773 | 0.9227 | 0 |
| 2000YS134 | 0.85736 | 0.2242 | 3.500566 | 0.8264 | 0.9981 | 0.7895 | 0 |
| 2001AF2 | 0.953982 | 0.5953 | 17.8175 | 0.0692 | 0.7104 | 0.6852 | 0.8186 |
| 2001BA16 | 0.940225 | 0.1374 | 5.768623 | 0.3057 | 0.9977 | 0.9806 | 0 |
| 2001BB16 | 0.854463 | 0.1723 | 2.026138 | 0.6033 | 0.9919 | 0.9797 | 0 |
| 2001BE10 | 0.823508 | 0.369 | 17.50827 | 1 | 0.9358 | 0.9119 | 0 |
| 2001CK32 | 0.725401 | 0.3826 | 8.137319 | 0.3553 | 0.9747 | 0.7198 | 0 |
| 2001CP36 | 0.714308 | 0.4077 | 10.53547 | 0.4545 | 0.9642 | 0.955 | 0 |

## Classification

- Data is available online
- verify and analyse data:

- Comparison with the *Project Spaceguard* (Milani et. al classification)
- Analysis with $\alpha$-cuts

http://www.celestialmechanics.eu/neas/

# Fuzzy NEA classification

• compare the classification data with the Spaceguard classes

<u>Geographos group</u>

• many close approaches with Earth

• some close approaches with Venus

• semimajor axis almost constant

<u>Oljato group</u>

• chaotic orbits

• large eccentricities

• close encounters with all planets

<u>(1620) Geographos:</u>

• G1 (no mixing): 1

• G2 (Venus): 0.03

• G3 (Earth): 0.76

<u>(2201) Oljato:</u>

• G1 (no mixing): 0.04

• G2 (Venus): 0.38

• G3 (Earth): 0.36

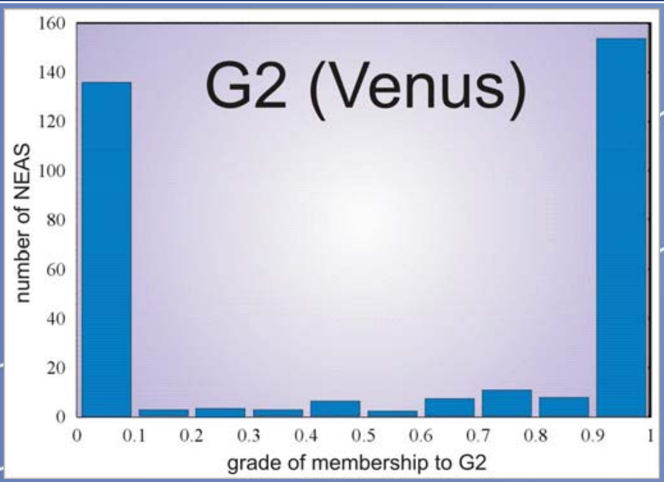
• G4 (Mars): 0.28

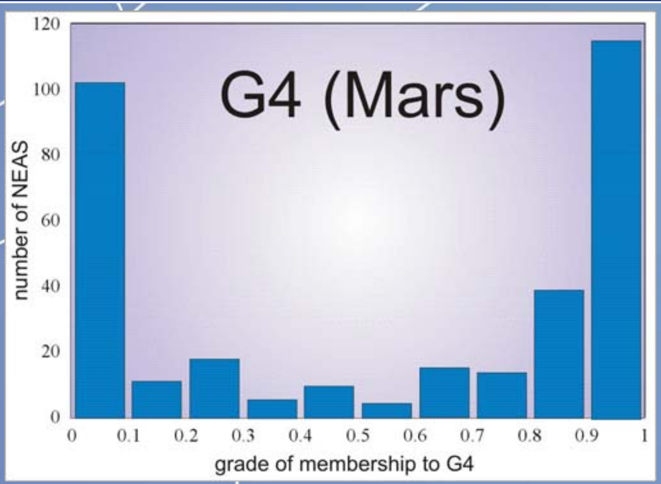
# $\alpha$-cut Analysis

- an $\alpha$-cut of a fuzzy set is a classical set:

$$A^{>\alpha} = \left\{ x \in X \middle| \mu_A(x) > \alpha \right\} \qquad \alpha \in [0,1]$$

- build an $\alpha$-cut and then investigate the distribution of grades of membership to the other classes
- e.g. $G3^{>0.9}$ : all asteroids that belong to G3 with more than 0.9
- all those asteroids also belong to G2 and G4 – with which degree ?




- either $G3^{>0.9}$ *and* $G2^{>0.9}$ or no membership to G2 at all
- same for G4 – but more „fluent“

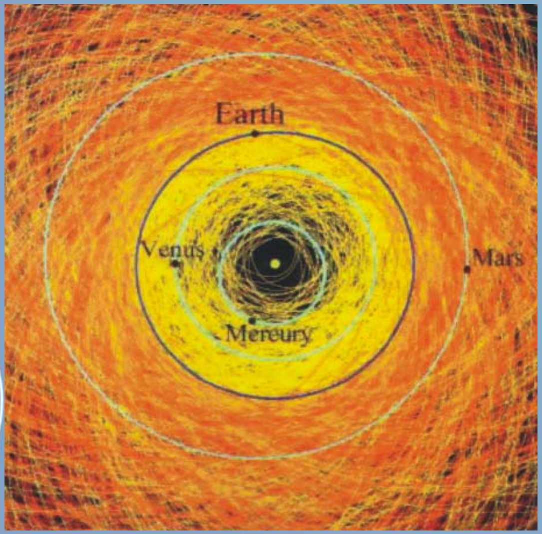
# Longterm fate of NEAs

• what happens with the NEAs?

• close encounters with planets → NEAs orbit everywhere between ~Mars and ~Venus

• → collision probability with Venus should be highest.

• ratio of mean motion of Venus and Earth: 1.38



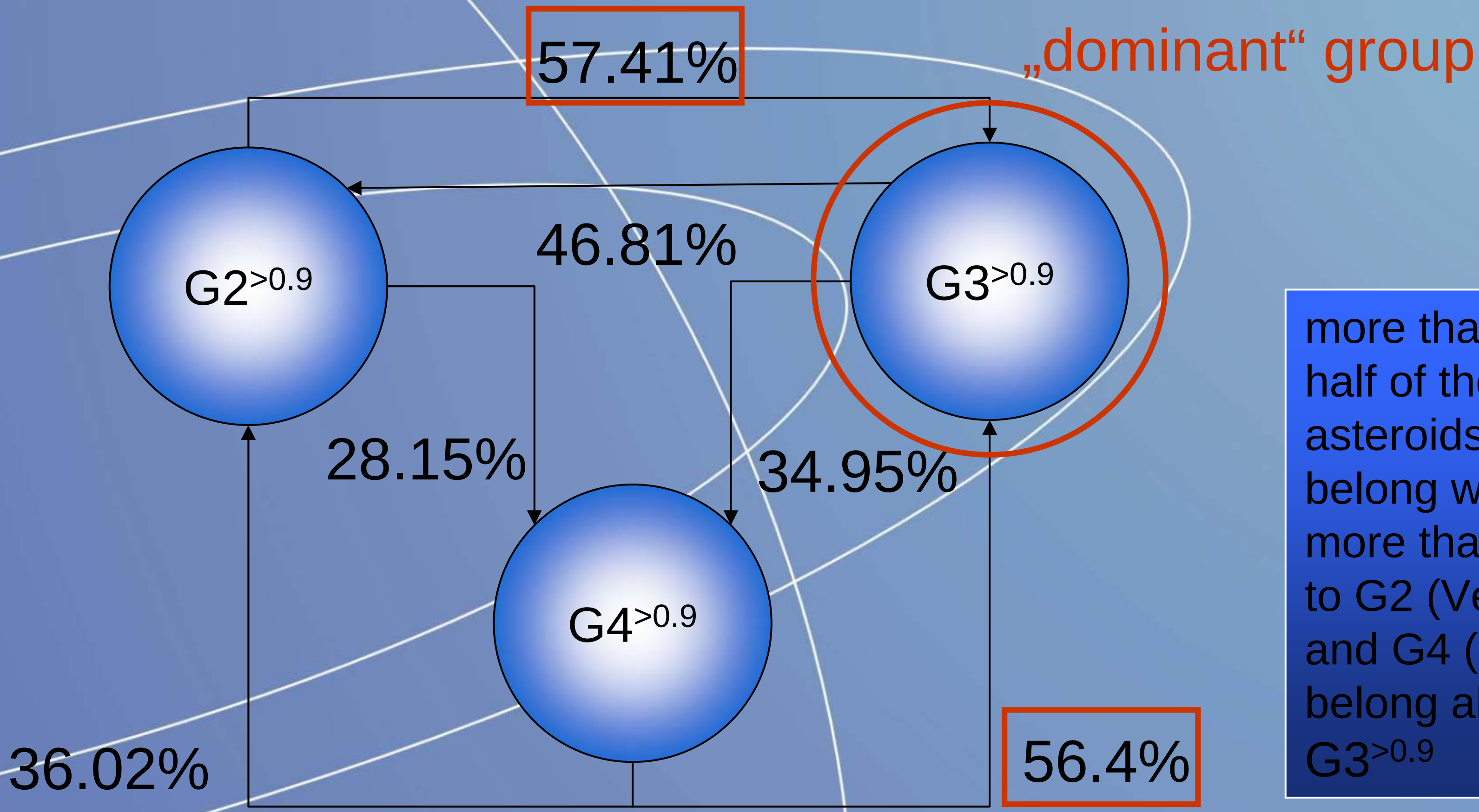

• ratio of collision probabilities of NEAs with Venus and Earth: ~1.3

• α-cut Analysis of G2, G3, G4:

• determine $G2^{>0.9}$, $G3^{>0.9}$, $G4^{>0.9}$ → investigate the mutual group membership

# Longterm fate of NEAs

- what happens with the NEAs?




more than the half of those asteroids that belong with more than 90% to G2 (Venus) and G4 (Mars) belong also to $G3^{>0.9}$

# Conclusions

- the chaoticity of NEA dynamics makes a statistical treatment difficult/impossible (mixing)
- fuzzy logic methods can overcome this problems
- a fuzzy characterisation of NEAs can incorporate the existing classifications (Project Spaceguard)
- using fuzzy groups allows a quantitative statistical analysis over long time scales
- the analysis has showed the importance of close encounters/collisions with Earth over long time scales

Future work

- new orbital data → refinement of membership functions
- define and analyse additional groups (resonances, …)

# THE END